Modelling Latent Constructs

Single Trait Models

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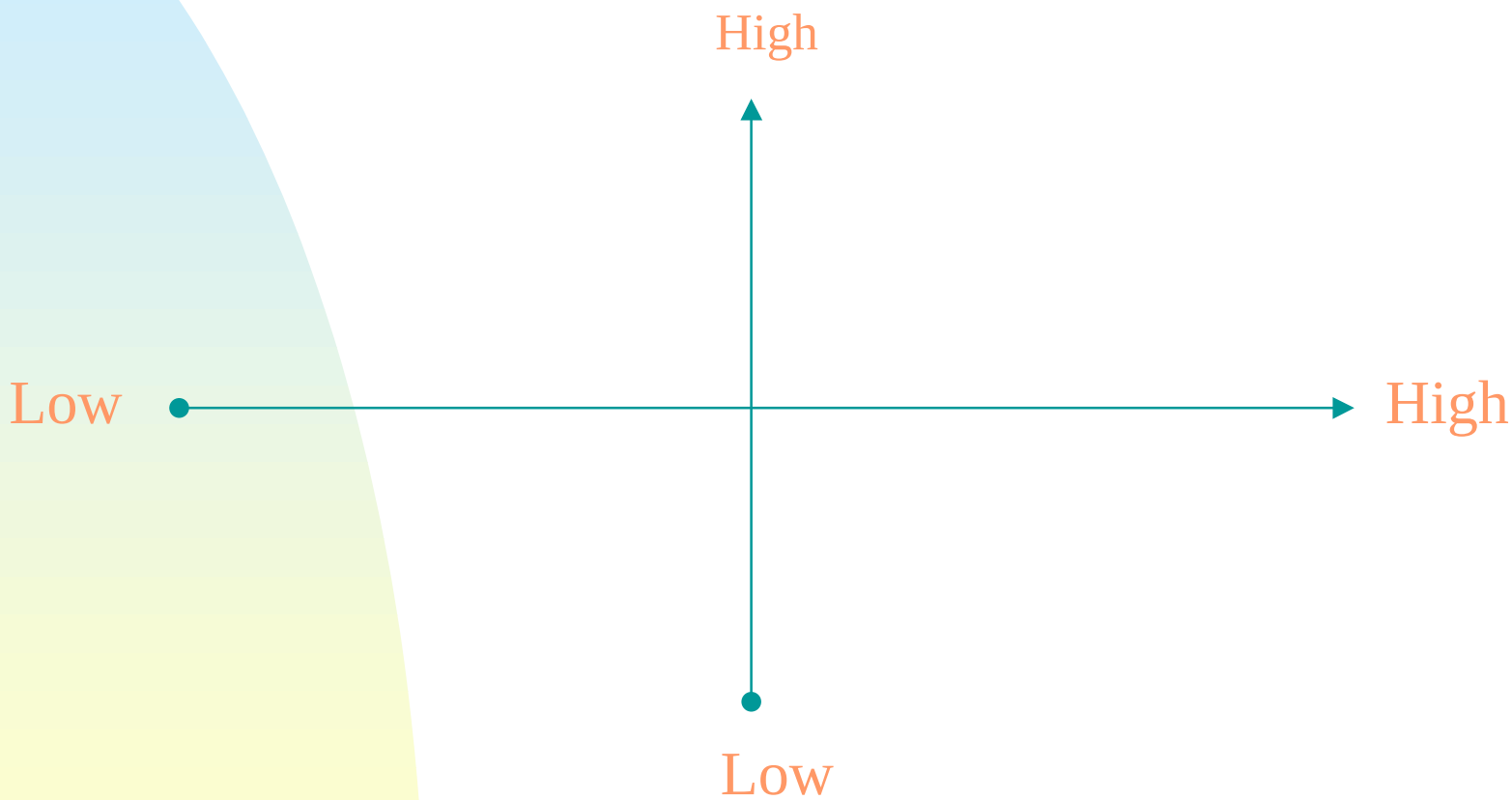
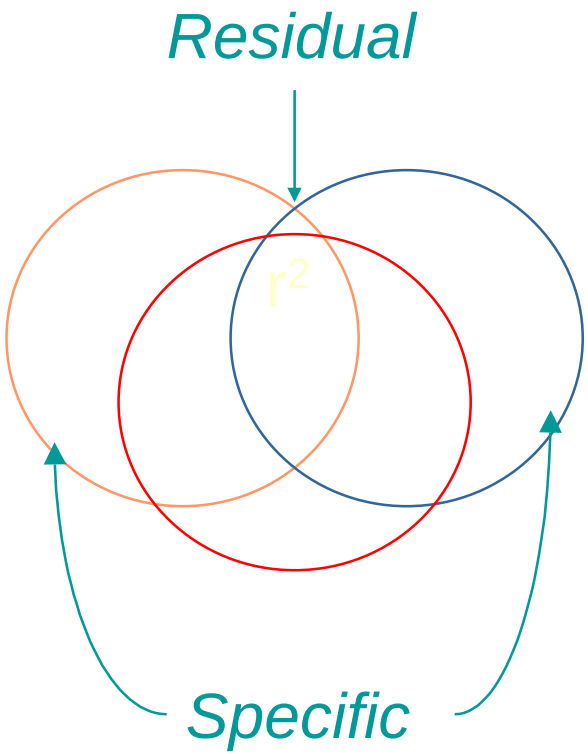
Types of Structure

- Discrete Clusters
- Hierarchical Clusters
- Spatial Configurations
- Underlying Latent Traits

Latent Traits



The Basics of Co-variation



Relationships Between Variables

	A	B	C	D	E	F
A	1.00					
B	0.74	1.00				
C	0.62	0.75	1.00			
D	0.50	0.61	0.72	1.00		
E	0.41	0.51	0.60	0.79	1.00	
F	0.35	0.45	0.50	0.69	0.72	1.00

All correlations are positive

All correlations are relatively large

An underlying simplex is indicated

The Simplex Structure

	A	B	C	D	E	F
A	1.00					
B	0.74	1.00				
C	0.62	0.75	1.00			
D	0.50	0.61	0.72	1.00		
E	0.41	0.51	0.60	0.79	1.00	
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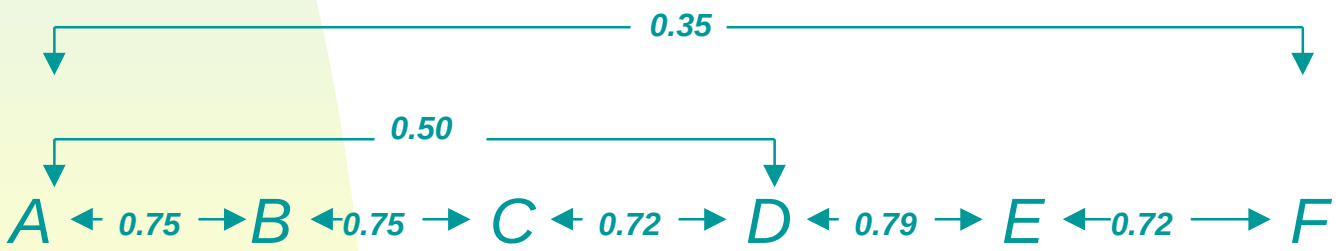

Homogeneity vs Structure

	A	B	C	D	E	F
A	1.00					
B	0.74	1.00				Alpha=0.90
C	0.62	0.75	1.00			
D	0.50	0.61	0.72	1.00		
E	0.41	0.51	0.60	0.79	1.00	
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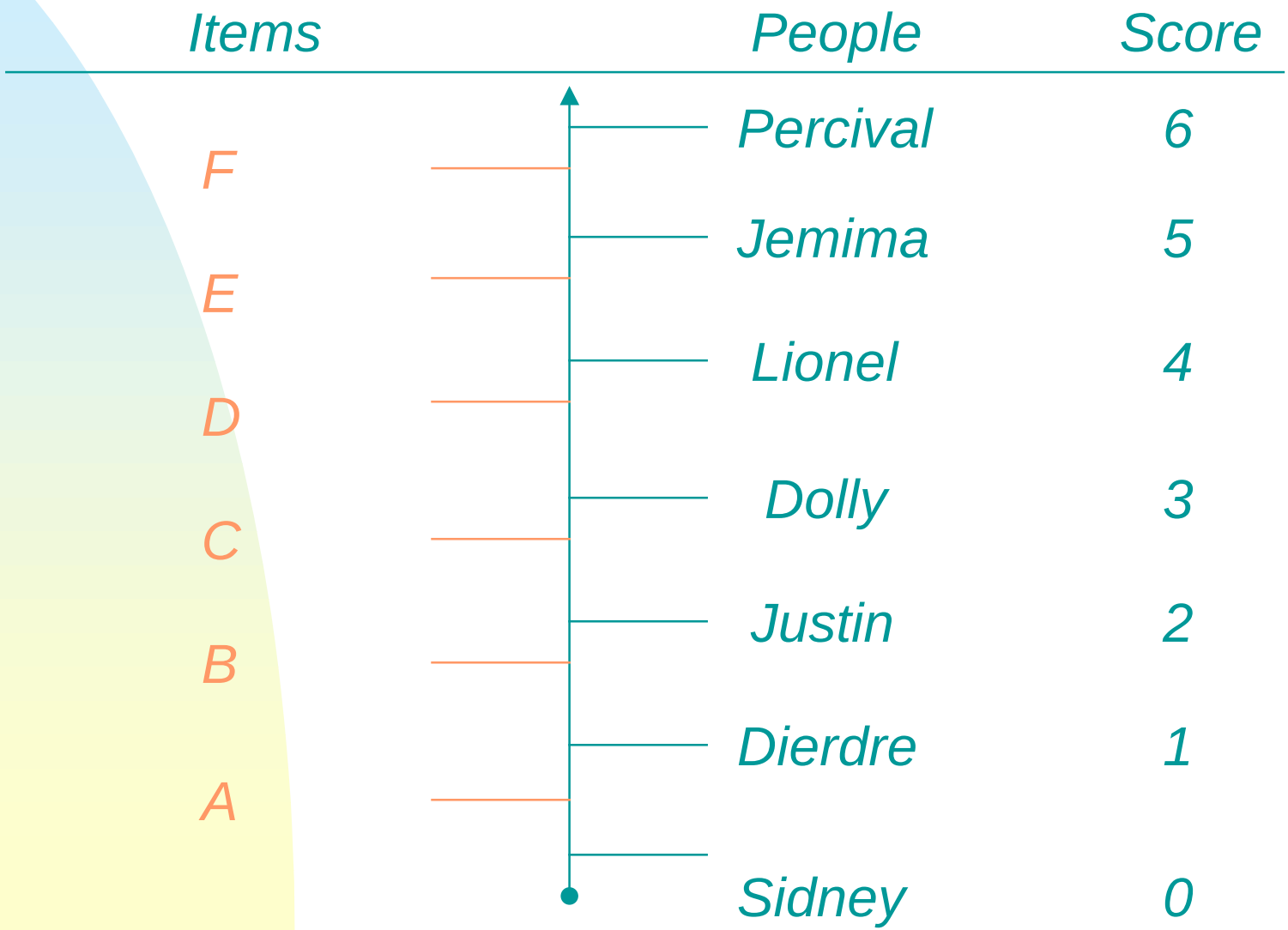
Unidimensional Structures

- Range from low to high
- Leave very little residual inter-variable covariation when partialed out
- Jointly orders the variables and the cases

The Unidimensional Scale

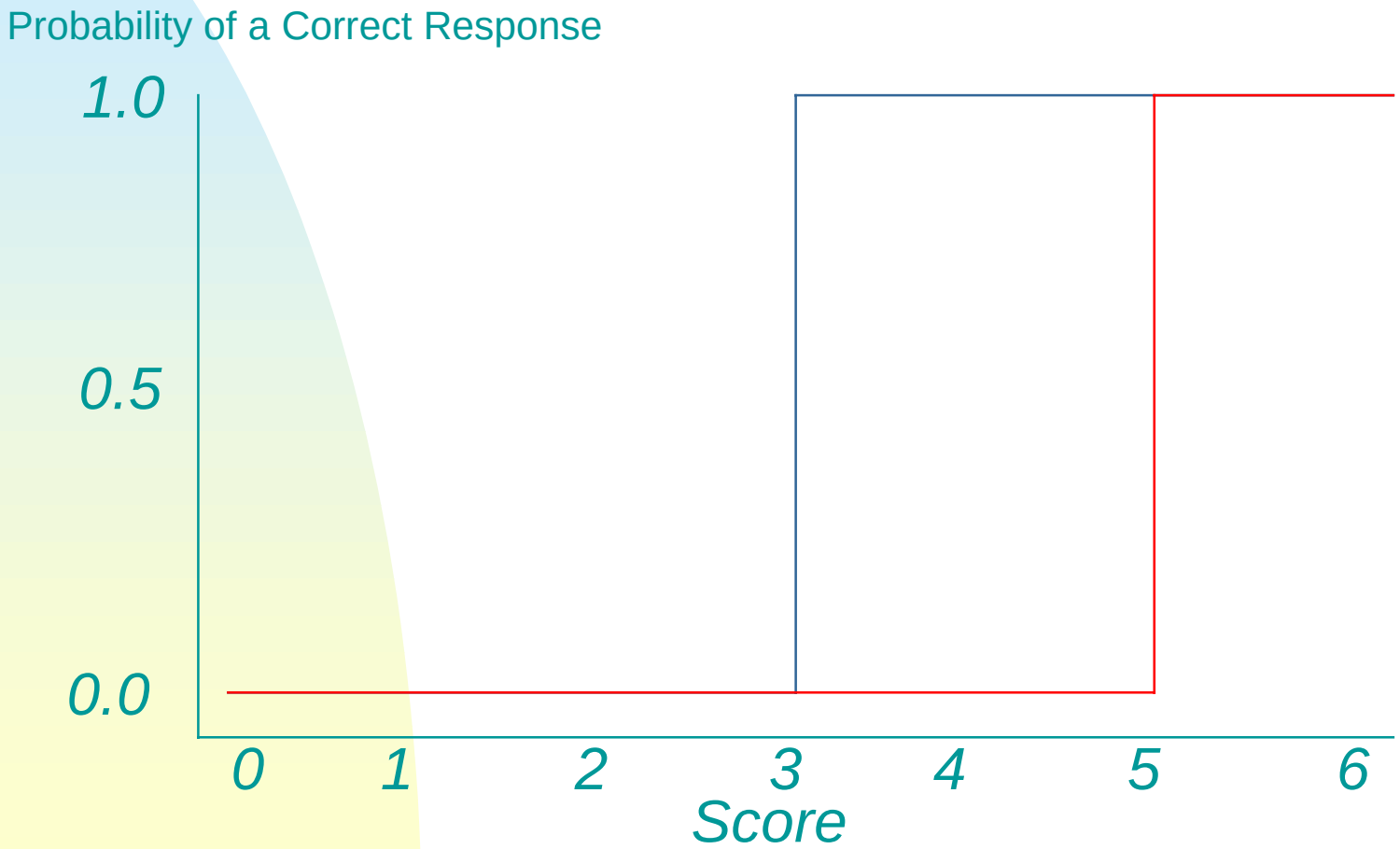


Profiles

Legitimate			Illegitimate		
Percival	111111	6	-		
Jemima	111110	5	Les	110111	5
Lionel	111100	4	Sirroco	011101	4
Dolly	111000	3	Bjorn	101001	3
Justin	110000	2	Caleb	000011	2
Dierdre	100000	1	Fanny	001000	1
Sidney	000000	0		-	

With 6 items scored dichotomously there are 128 (2^6) possible score profiles. Only seven (M+1) of them will legitimately fit the unidimensional scale.

The Item Characteristic Curve



Fitting The Guttman Scalogram

- For each profile identify the 'errors'
- Sum all the errors
- Derive the ratio of errors to responses

Reproducability Coefficient

$$R = 1 - \frac{E}{(N*M)}$$

Example: Green Affiliation Scale

A13	Take part in conservation projects.
A14	Learn about the issues.
A15	Join environmental campaigns.
A16	Go to environmental group meetings.
A17	Vote on environmental grounds.
A18	Write protest letters.
A19	Try to convert friends/acquaintances.
A20	Read environment group circulars.
Sup	Support the Green party.
Vote	Vote for the Green party.
Prog	Seek out environmental programmes on TV.
Memb	Become a member of a green group.
Spec	Buy specialist environmental magazines and papers.

Item Parameters

Item Name	% on 0	% on 1	Difficulty
A14	42.192	57.808	0.578
SUP	44.658	55.342	0.553
PROG	51.233	48.767	0.488
A20	60.274	39.726	0.397
A17	65.205	34.795	0.348
A19	65.479	34.521	0.345
SPEC	78.082	21.918	0.219
A13	80.274	19.726	0.197
A15	82.466	17.534	0.175
A18	84.932	15.068	0.151
MEMB	90.411	9.589	0.096
A16	91.781	8.219	0.082
VOTE	94.247	5.753	0.058

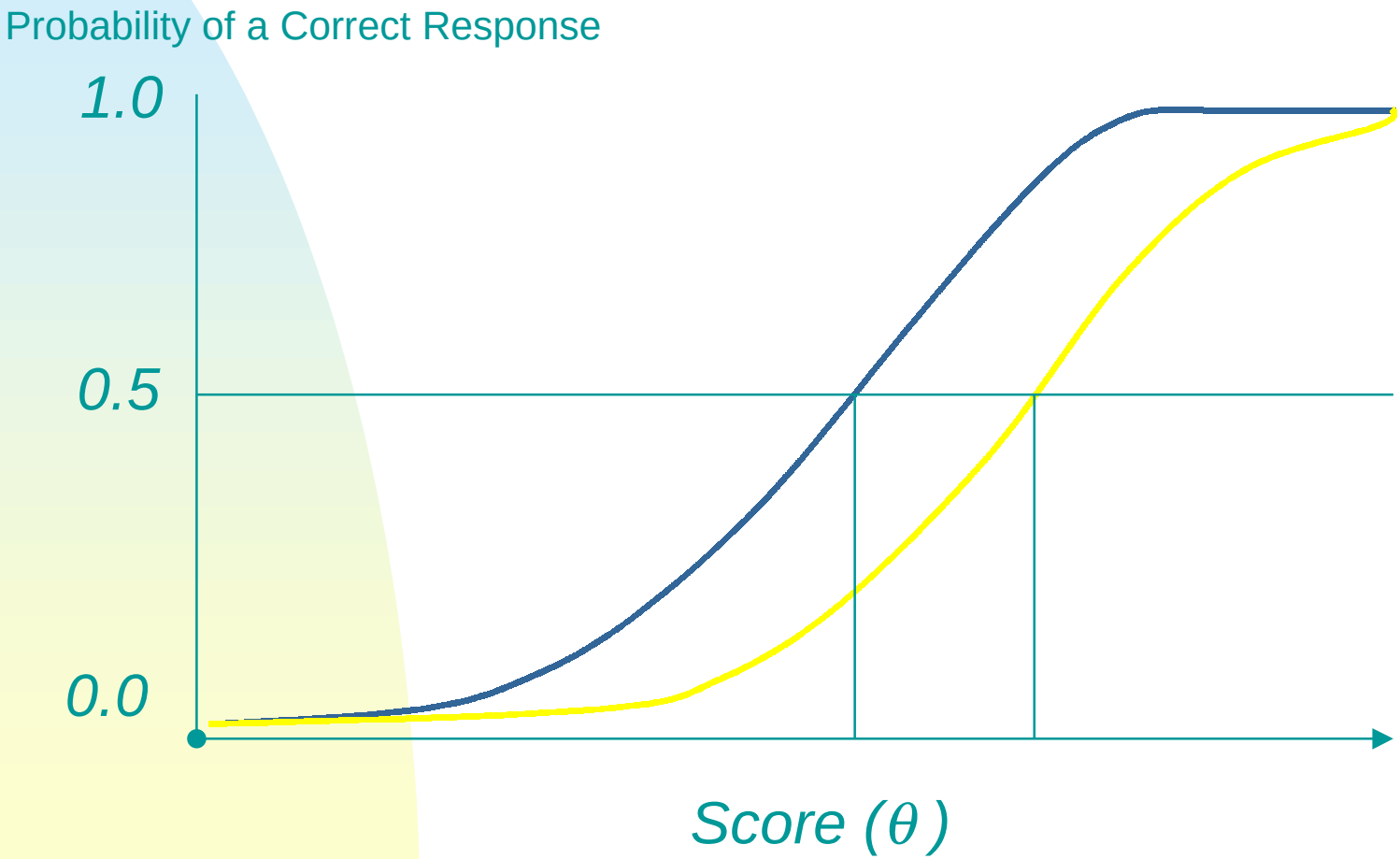
Fit of the Scalogram Model

Number of possible profiles	= 8192
Number of observed profiles	= 176
Ratio of observed to possible profiles	= 0.021
Coefficient of Reproducability	= 0.670
Minimal Marginal Reproducability	= 0.737
Scalability Coefficient	= 0.622

Limitations with Scalograms

- Impractical with large item pools
- Very little available software
- Deterministic Model

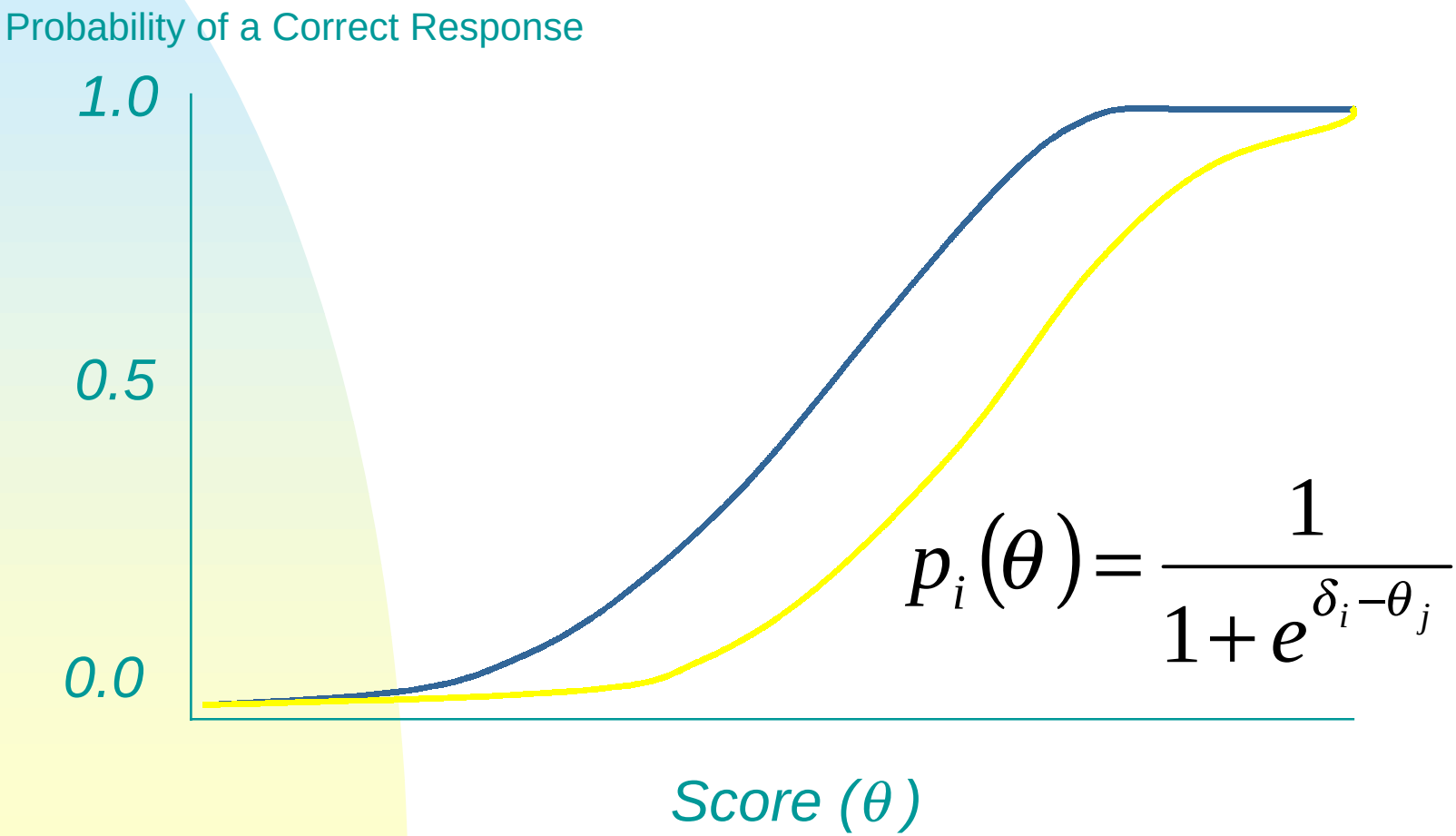
Probabilistic ICC



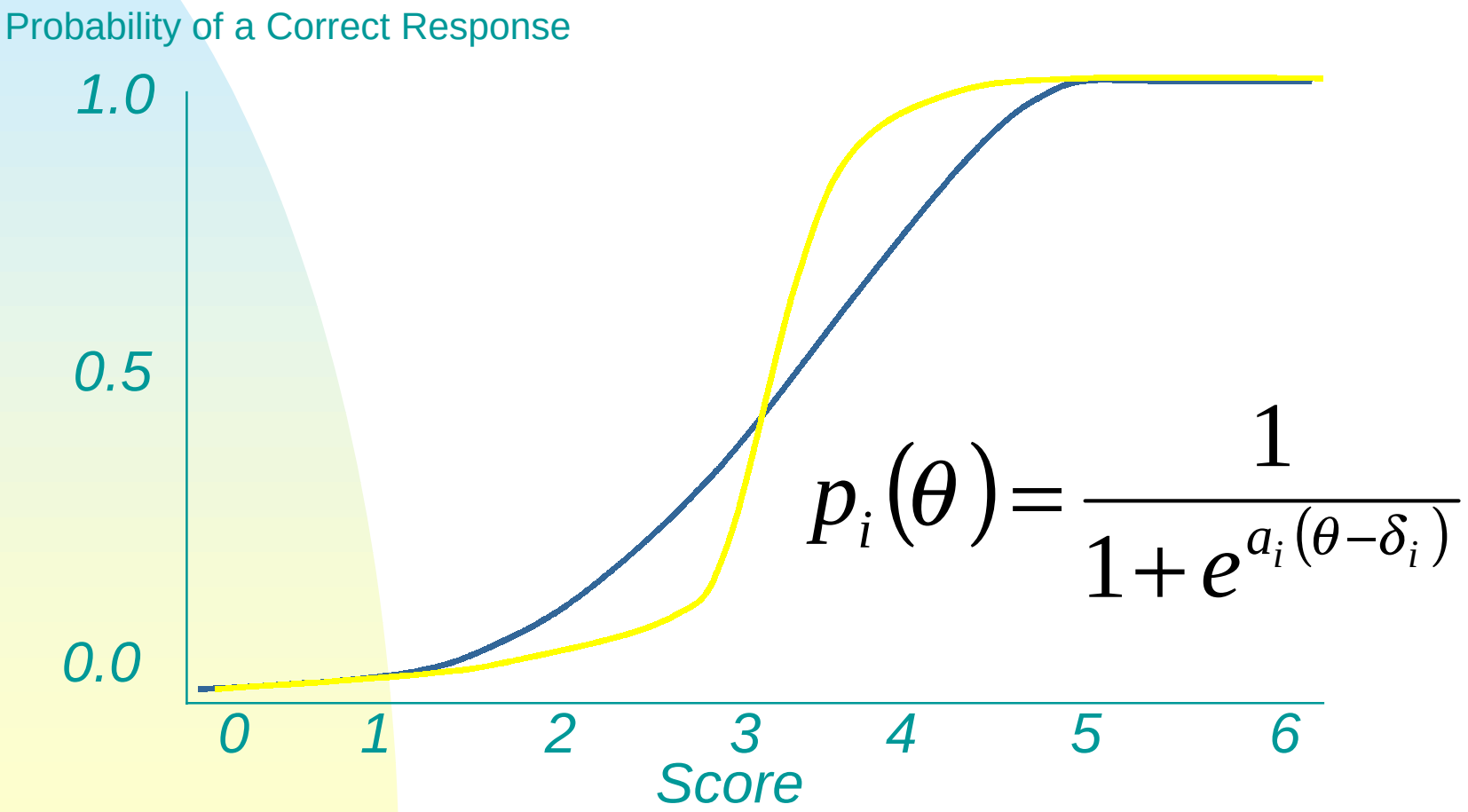
IRT Models

- Mokken (non parametric)
- Rasch (1 parameter) Difficulty
- Birnbaum (2 parameter) Discrimination
- Lord (3 parameter) Guessing

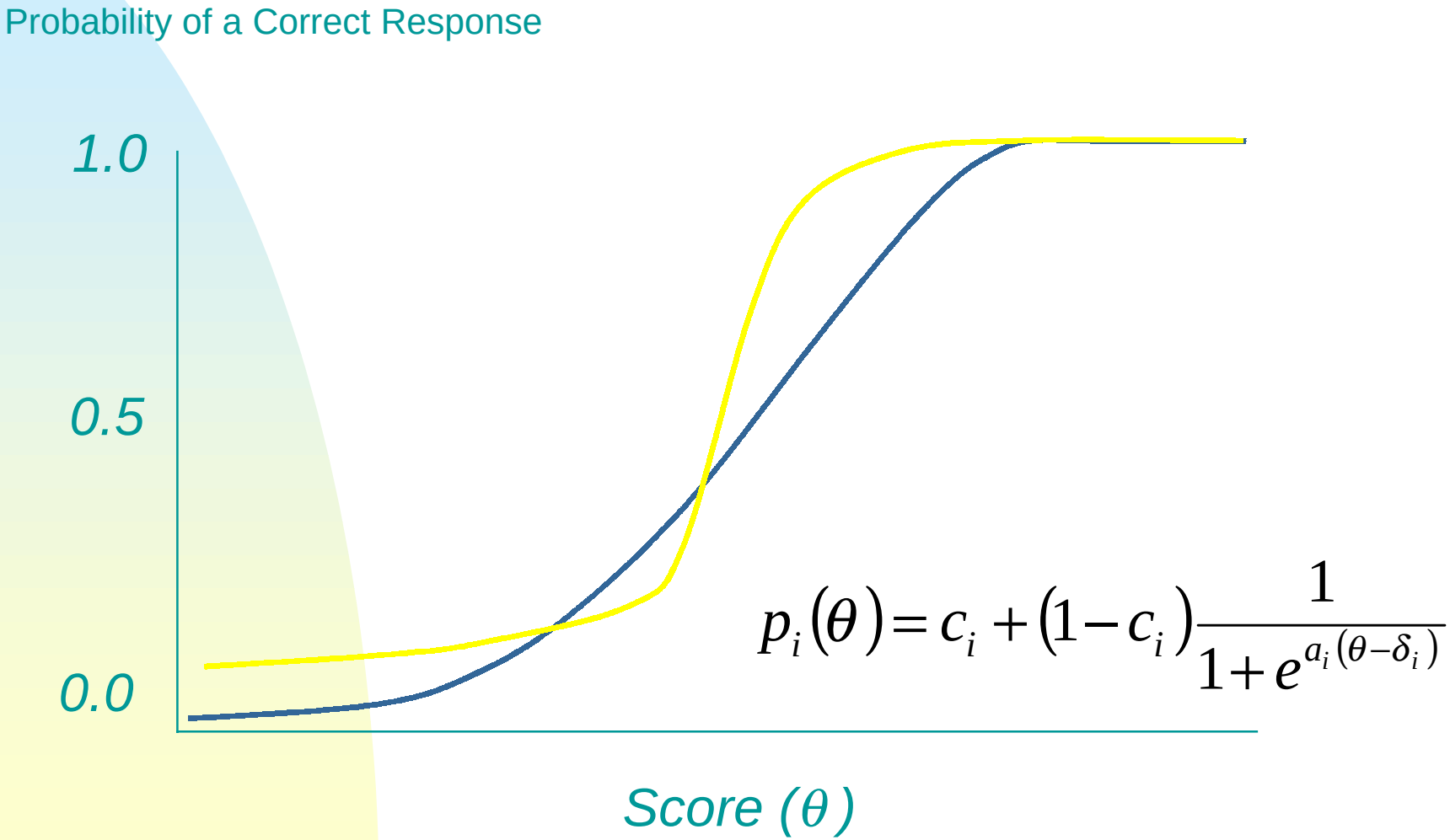
1 Parameter ICC



2 Parameter ICC



3 Parameter ICC



The Rasch Model

- Cumulative Scale
- Additive Conjoint Measurement
- Simple Interpretation

Rasch Model Fit

Item Name	Mean	δ	S.E.	z Ratio	p
A14	0.578	-4.272	0.315	0.854	0.2343
SUP	0.553	-3.974	0.215	1.102	0.1721
PROG	0.488	-3.190	0.049	2.034	0.0349
A20	0.397	-2.099	0.417	0.850	0.2611
A17	0.348	-1.472	0.628	0.288	0.2101
A19	0.345	-1.436	0.640	0.440	0.5001
SPEC	0.219	0.436	0.270	0.776	0.6125
A13	0.197	0.831	0.404	0.470	0.4984
A15	0.175	1.260	0.548	0.304	0.4052
A18	0.151	1.798	0.729	0.278	0.2662
MEMB	0.096	3.325	0.244	1.063	0.2011
A16	0.082	3.828	0.413	0.970	0.7347
VOTE	0.058	4.966	0.796	1.992	0.0448

Reliability = 0.83

Overall Fit

$X^2 = 14.32$

$Df = 12$

What IRT Allows Us To Do

- Fit a test to a strict unidimensional model
- Find a person's position on a latent trait
- Identify misfit in the items
- Identify misfit in the people
- Generate adaptive tests
- Develop axiomatic measurement

The Growing Literature

- Indices of fit
- Methods of Estimation
- Model Specification